AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST RANKER'S BATCH-01 TEST - 03 Balliwala SOLUTION Date 22-09-2019

[PHYSICS]

1. $l = \frac{FL}{AY} \therefore l \propto \frac{1}{r^2}$ (F, L and Y are constant)

$$\frac{l_1}{l_2} = \left(\frac{r_2}{r_1}\right)^2 = (2)^2 = 4$$

2. $l = \frac{FL}{\pi r^2 Y} \Rightarrow r^2 \propto \frac{1}{Y}$ (F, L and l are constant)

$$\frac{r_2}{r_1} = \left(\frac{Y_1}{Y_2}\right)^{1/2} = \left(\frac{7\times 10^{10}}{12\times 10^{10}}\right)^{1/2}$$

$$\Rightarrow \quad r_2 = 1.5\times\left(\frac{7}{12}\right)^{1/2} = 1.145 \text{ mm} \therefore \text{dia} = 2.29 \text{ mm}$$

3. $F = Y\times A\times\frac{l}{L} \Rightarrow F \propto \frac{r^2}{L}$ (Y and l are constant)

$$\frac{F_A}{F_B} = \left(\frac{r_A}{r_B}\right)^2\times\left(\frac{L_B}{L_A}\right) = \left(\frac{2}{1}\right)^2\times\left(\frac{2}{1}\right) = \frac{8}{1}$$

4. Young's modulus depends upon the nature of material and not the radii of the wires.

5. Breaking stress does not depend upon the length of the cable.

6. Stress = $Y\times$ Strain

$$= 2\times 10^{11}\times 0.15 \text{ N m}^{-2} = 3\times 10^{10} \text{ N m}^{-2}$$

7. $Y = \frac{Fl}{A\Delta l}$ or $F = \frac{YA\Delta l}{l}$

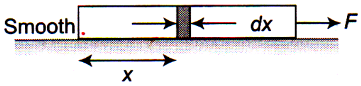
or $F = \frac{2.2\times 10^{11}\times 2\times 10^{-6}\times 0.5\times 10^{-3}}{2} = 1.1\times 10^2\text{N}$

8. The tension, T in the rope of the lift when it goes upward is given by

9. $K = \frac{\Delta p}{\Delta V/V} = \frac{(1.165-1.01)\times 10^5}{10/100} = \frac{0.155\times 10^5}{1/10}$

$$= 1.55\times 10^5 \text{ pa}$$

10 Tension, $T = \frac{F}{L_0} \cdot x x$



Stress, $\sigma = \frac{T}{A} = \frac{F}{AL_0}x$

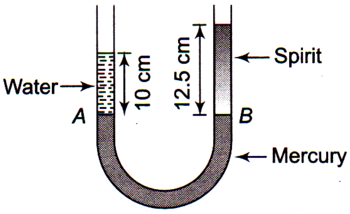
$$dU = \frac{1}{2} \cdot \frac{\sigma^2}{Y} Adx = \frac{1}{2} \frac{F^2}{A^2 L_0^2} \cdot x^2 \frac{A}{Y} dx$$

or $$dU = \frac{F^2}{2A^2 L_0^2 Y} \cdot x^2 dx$$

$\Rightarrow$ $$U = \frac{F^2}{2AY L_0^2} \int_0^{L_0} x^2 dx$$

$$U = \frac{F^2}{2AY L_0^2} \cdot \frac{L_0^3}{3} = \frac{F^2 L_0}{6AY}$$

11 As the mercury columns in the two arms of U tube are at the same level, therefore



Pressure due to water = Pressure due to spirit column

$$\rho w_h w_g = \rho_s h_s g$$

$$\rho_s = \frac{h_w}{h_s} \rho_w$$

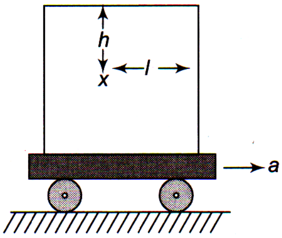
$\therefore$ Relativedensityofspirit $= \frac{\rho_s}{\rho_w} = \frac{h_w}{h_s} = \frac{10 \text{ cm}}{12.5 \text{ cm}} = 0.8$

12. Pressure = $h\rho g$
Pressure at the bottom is independent of the area of the base of the vessel. It depends on the height of water upto which the vessel is filled with water. As in all the three vessels, level of water are the same, therefore Pressure at the bottom in all the vessels is also same.
Hence, $P_A = P_B = P_C$

13. $\tan\theta = \frac{3b-b}{4b} = \frac{a}{g} \Rightarrow a = \frac{g}{2}$

14. The pressure P at point x in the sum of pressure P_1 and P_2 where P_1 is the pressure due to gravity and P_2

is the pressure necessary to impart an acceleration a to the column of water of length l. Pressure P_2 acting on a column of length l and area of cross–section A gives it an acieration a in the horizontal direction.



$P_2A = Alda$

$P_2 = lda$

As pressure is a scalar quantity, so

$P = P_1 + P_2 = hdg + lda = d(hg + la)$

15.

$V_D = \frac{m_C}{\rho_w} + \frac{m_B}{\rho_w}$ and $V_g = \frac{m_C}{\rho_C} + \frac{m_B}{\rho_w}$

Since $\rho_C > \rho_w$, $V_g < V_D$

Hence, l and h both decrease.

16.

At point P, $P_P = 0$

At point Q, $P_Q = P_P + \rho gh = \rho gh$

At point R, $P_R = P_Q + \rho aL = \rho gh + \rho aL$

At point S, $P_S = P_R - \rho gh = \rho aL$

17.

Weight of cylinder = upthrust to both liquids

$V \times D \times g = \left(\frac{A}{5} \times \frac{3}{4}L\right) \times d \times g + \left(\frac{A}{5} \times \frac{L}{4}\right) \times 2d \times g$

or, $\left(\frac{A}{5} \times L\right) \times D \times g = \frac{A \times L \times d \times g}{4}$

or, $\frac{D}{5} = \frac{d}{4}$ or $D = \frac{5}{4}d$

18.

The velocity of ball before entering the water surface

$v = \sqrt{2gh} = \sqrt{2g \times 9}$

When ball enters into water, due to upthrust of water, the velocity of ball decreases (or retarded)

The retardation $a = \frac{\text{apparent weight}}{\text{mass of ball}}$

$= \frac{V(\rho - \sigma)}{V\rho} = \left(\frac{\rho - \sigma}{\rho}\right)g = \left(\frac{0.4 - 1}{0.4}\right) \times g = -\frac{3g}{2}$

If h be the depth upto which ball sinks then

$0 - v^2 = 2 \times \left(-\frac{3}{2}g\right) \times h$ or $2g \times 9 = 3\,gh$

$\therefore h = 6$ cm

19.

Specific gravity of alloy $= \frac{\text{density of alloy}}{\text{density of water}}$

$= \frac{\text{mass of alloy}}{\text{volume of alloy} \times \text{density of water}}$

$= \frac{m_1 + m_2}{\left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}\right) \times \rho_w} = \frac{m_1 + m_2}{\left(\frac{\frac{m_1}{\rho_1}}{\rho_w} + \frac{\frac{m_2}{\rho_2}}{\rho_w}\right)}$

$= \frac{m_1 + m_2}{\left(\frac{m_1}{s_1} + \frac{m_2}{s_2}\right)}$

20.

$\rho_1 < \rho_2$ as denser liquid acuires lowest position of vessel. $\rho_3 > \rho_1$ as ball sinks in liquid 1 and $\rho_3 < \rho_2$ as ball doesn't sinks in liquid 2, so

$\rho_1 < \rho_3 < \rho_2$

21.

Pressure at same horizontal level is same.

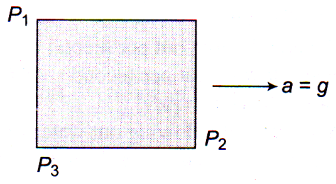
$P + \rho_1 g h_1 = P_0 + \rho_2 g h_2$

$P - P_0 = g(\rho_2 h_2 - \rho_1 h_1)$

22. $\tan\theta = \frac{a}{g} = \frac{h_2 - h_1}{h_2 \tan 45° + h_1 \tan 45°} = \frac{4\text{ cm}}{20\text{ cm}}$

$\Rightarrow \quad a = 2\text{ m/s}^2$

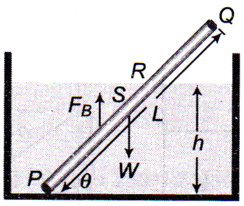
23 Let pressure at the opening be P_o. Then,
$P_1 = P_o + \rho g b$ while $P_2 = P_o + \rho g h$.



Subtracting, $P_1 - P_2 = \rho g(b - h)$

24. Tension in spring T = upthrust – weight of sphere
$= V\sigma g - V\rho g = V\eta\rho g - V\rho g$ (As $\sigma = n\rho$)
$= (\eta - 1)V\rho g = (\eta - 1)mg$

25.



Let $L = PQ$ = length of rod

$\therefore \quad SP = SQ = \dfrac{L}{2}$

Weight of rod, $W = AL\rho g$, acting
At point S
And force of buoyancy,

$F_B = Al\rho_0 g$, $[l = PR]$

which acts at mid-point of PR.
For rotational equilibrium,

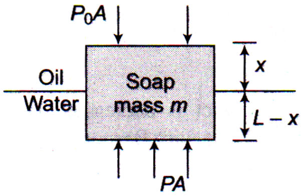
$$Al\rho_0 g \times \frac{l}{2}\cos\theta = AL\rho g \times \frac{L}{2}\cos\theta$$

$$\Rightarrow \quad \frac{l^2}{L^2} = \frac{\rho}{\rho_0} \Rightarrow \frac{l}{L} = \sqrt{\frac{\rho}{\rho_0}}$$

From figure, $\sin\theta = \dfrac{h}{l} = \dfrac{L}{2l} = \dfrac{1}{2}\sqrt{\dfrac{\rho_0}{\rho}}$

26. Fraction of volume immersed in the liquid $V_{in} = \left(\dfrac{\rho}{\sigma}\right)V$ i.e. it depends upon the densities of the block and liquid.
So there will be no change in it if system moves upward or downward with constant velocity or some acceleration.

27. Let A is area of soap bar.



$$PA - P_0A = mg \Rightarrow (P - P_0)A = mg$$

$\Rightarrow$ $[300gx + 1000g(L-x)A = AL800g$

$\Rightarrow$ $\frac{x}{L} = \frac{2}{7}$

28 $$t = \frac{A}{a}\sqrt{\frac{2}{g}}[\sqrt{H_1} - \sqrt{H_2}]$$

Now $T_1 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{\frac{H}{\eta}}\right]$

and $T_2 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{\eta}} - \sqrt{0}\right]$

According to problem $T_1 = T_2$

$\therefore$ $\sqrt{H} - \sqrt{\frac{H}{\eta}} = \sqrt{\frac{H}{\eta}} - 0 \Rightarrow \sqrt{H} = 2\sqrt{\frac{H}{\eta}} \Rightarrow \eta = 4$

29. Pressure at the bottom of tank $P = h\rho g = 3 \times 10^5 \frac{\text{N}}{\text{m}^2}$
Pressure due to liquid column

$$P_l = 3 \times 10^5 - 1 \times 10^5 = 2 \times 10^5$$

and velocity of water $v = \sqrt{2gh}$

$\therefore$ $v = \sqrt{\frac{2P_l}{\rho}} = \sqrt{\frac{2 \times 2 \times 10^5}{10^3}} = \sqrt{400}$ m/s

30. When the container is at rest with respect to the Earth, there is pressure on the walls due to the weight of the water. The pressure results from the contact force between the water and the container. In free fall, both the water and the container have acceleration of g, and the contact force is zero, so removing part of a wall by making a hole produces no outward flow. (Note that some of the water is in contact with the air, which is not accelerating, so there is still atmospheric pressure on the water.)

31. Required ratio is 1/3

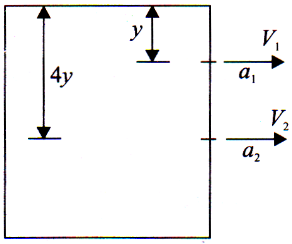
32. Let the density of water be ρ, then the force by escaping liquid on container = $\rho S(\sqrt{2gh})^2$
Acceleration of container

$$a = \frac{2\rho Sgh - \mu\rho Vg}{\rho V} = \left(\frac{2Sh}{V} - \mu\right)g$$

Now $\mu = \frac{Sh}{V}$ $\therefore$ $a = \frac{Sh}{V}g$

33. Velocity of efflux at a depth h is given by

$$v = \sqrt{2gh}$$



Volume of water flowing out per second from both the holes are equal.

$\therefore \quad a_1 v_1 = a_2 v_2$

or $\quad L^2\sqrt{2gy} = \pi R^2 \sqrt{2g(4y)}$

or $\quad R = \dfrac{L}{\sqrt{2\pi}}$

34. $0.18\, v + 0.12 \times 1.5 = 0.12 \times 3$

or $\quad 0.18\, v = 0.12 \times 1.5$

or $\quad v = \dfrac{0.12 \times 1.5}{0.18} \text{ms}^{-1} = 1 \text{ ms}^{-1}$

35. Maas of liquid in horizontal portion of U-tube $= A\, d\rho$

Pseudo force on this mass $= Ad\rho a$

Force due to pressure difference in the two limbs

$= (h_1\rho g - h_2\rho g)A$

Equating, $(h_1 - h_2)\rho g A = Ad\rho a$

or $\quad h_1 - h_2 = \dfrac{Ad\rho a}{\rho g A} = \dfrac{ad}{g}$

36. Pressure at the bottom of tank $P = h\rho g = 3 \times 10^5 \dfrac{N}{m^2}$.

Pressure due to liquid column $P_l = 3 \times 10^5 - 1 \times 10^5 = 2 \times 10^5$ and velocity of water $v = \sqrt{2gh}$

$\therefore \quad v = \sqrt{\dfrac{2P_l}{\rho}} = \sqrt{\dfrac{2 \times 2 \times 10^5}{10^3}} = \sqrt{400}$ m/s

37. Effective value of acceleration due to gravity becomes $(g + a_0)$.

38. $x=\sqrt{2gh_1}\times\sqrt{\frac{2h_2}{g}}$ or $x=2\sqrt{h_1h_2}$

Now, imagine a hole at a depth h_2 below the free surface of the liquid. The height of this hole will be h_1. Clearly, x remains the same.

39. $v=\sqrt{2gh}$

But $p=h\rho g$ or $\frac{p}{\rho}=gh$

$\therefore$ $v\sqrt{\frac{2p}{\rho}}=\sqrt{\frac{2\times2\times10^5}{10^3}}\ \text{ms}^{-1}=20\ \text{ms}^{-1}$

$R^2v=\text{constant}$

40. From Torricelli's theorem

$$v=\sqrt{2gd} \qquad \text{(i)}$$

where v is horizontal velocity and d is the depth of water in barrel.

Time t to hit the ground is given by

$$h=\frac{1}{2}gt^2 \quad \text{or} \quad t=\sqrt{\frac{2h}{g}}$$

$\therefore$ $R=vt=\sqrt{(2gd)}\sqrt{\frac{2h}{g}}=2\sqrt{dh}$ (Using (i))

$\therefore$ $R^2=4dh$ or $d=\frac{R^2}{4h}$

41. $4(H-4)=6(H-6)$

or $2H=36-16-20$ or $H=10$ cm

42. Let A and a be the cross-sectional areas of the vessel and hole respectively. Let h be the height of water in the vessel at time. Let $\left(-\frac{dh}{dt}\right)$ represent the rate of fall of level.

Then, $A\left(-\frac{dh}{dt}\right) = \alpha v = a\sqrt{2gh}$

or $-\frac{dh}{\sqrt{h}} = \frac{\alpha\sqrt{2g}}{A}dt$

$$-\int_A^0 \frac{1}{\sqrt{h}}dh = \frac{a\sqrt{2g}}{A}\int_0^g dt$$

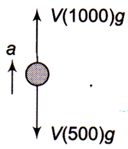
$$-(-2\sqrt{h}) = \frac{\alpha\sqrt{2g}}{A}t$$

or $t = \frac{A}{\alpha}\frac{1}{\sqrt{2g}} \times 2\sqrt{h}$ or $t = \frac{A}{\alpha}\sqrt{\frac{2h}{g}}$

Now, $t \propto \sqrt{h}$

When h is quadrupled, t is doubled.

43. Velocity of ball when it reaches to surface of liquid



$a = \frac{1000\text{ gV} - 500\text{ gV}}{500\text{ V}}$; where V is the volume of the ball.

$a = 10\text{ m/sec}^2$

Apply $v = u + at \Rightarrow 0 = \sqrt{2gh} - 10t$

$\Rightarrow \sqrt{2gh} = 10 \times (2)$

$\Rightarrow 2 \times 10 \times h = 400 \Rightarrow = 20\text{ m}$

44. Velocity u of the body when it enters the liquid is given by $mgh = \frac{1}{2}mu^2$ or $u = \sqrt{2gh}$

Let Volume of the body = V

Mass of the body = Vd

Weight of the body = Vdg

Mass of liquid displaced = VD

Weight of liquid displaced = VDg

Net upward force = $VDg - VDg$

$= V_g(D - d)$

$$\text{Retardation} = \frac{\text{net weight}}{\text{mass}}$$

$$= \frac{V(D-d)g}{Vd} = \left(\frac{D-d}{d}\right)g$$

Acceleration $a = -\left(\frac{D-d}{d}\right)g$

Final velocity, v in the liquid when the body is instantaneously at rest is zero. Let the time taken be t.

$$v = u + at$$

$$0 = \sqrt{2gh} - \left(\frac{D-d}{d}\right)gt \cdot \left(\frac{D-d}{d}\right)gt = \sqrt{2gh}$$

$$\therefore \quad t = \left[\frac{d}{D-d}\right]\sqrt{\frac{2h}{g}}$$

45. The weight of the aircraft is balanced by the upward force due to the Pressure difference.

i.e.,

$$\Delta P = \frac{mg}{A} = \frac{(4 \times 10^5 \text{ kg})(10 \text{ ms}^{-2})}{500 \text{ m}^2} = \frac{4}{5} \times 10^4 \text{ N m}^{-2}$$

$$= 8 \times 10^3 \text{ N m}^{-2}$$

Let v_1, v_2 are the speed of air on the lower and upper surface of the wings of the aircraft and P_1, P_2 are the pressures there.

Using Bernoulli's theorem, we get

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}(\rho v_2^2 - \rho v_1^2)$$

$$\Delta P = \frac{\rho}{2}(v_2 + v_1)(v_2 - v_1)$$

or $\quad v_2 - v_1 = \frac{\Delta P}{\rho v_{av}}$